

09-05-07

DTU



Attorney's Docket No. 042933/298965

PATENT

**IN THE UNITED STATES PATENT AND TRADEMARK OFFICE**

Inventor: Nativadade Lobo  
Appl. No.: 09/625,201  
Filed: July 21, 2000  
For: PULSE SHAPING WHICH COMPENSATES FOR COMPONENT DISTORTION

Confirmation No.: 5615

BOX ISSUE FEE  
Commissioner for Patents  
P.O. Box 1450  
Alexandria, VA 22313-1450

**SUBMITTAL OF PRIORITY DOCUMENT**

To complete the requirements of 35 U.S.C. § 119, enclosed is a certified copy of Great Britain priority Application No. 9804600.6, filed March 5, 1998.

Respectfully submitted,

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I, the undersigned, being an officer duly authorised in accordance with Section 74(1) and (4) of the Deregulation & Contracting Out Act 1994, to sign and issue certificates on behalf of the Comptroller-General, hereby certify that annexed hereto is a true copy of the documents as originally filed in connection with patent application GB9804600.6 filed on 5 March 1998.

In accordance with the Patents (Companies Re-registration) Rules 1982, if a company named in this certificate and any accompanying documents has re-registered under the Companies Act 1980 with the same name as that with which it was registered immediately before re-registration save for the substitution as, or inclusion as, the last part of the name of the words "public limited company" or their equivalents in Welsh, references to the name of the company in this certificate and any accompanying documents shall be treated as references to the name with which it is so re-registered.

In accordance with the rules, the words "public limited company" may be replaced by p.l.c., plc, P.L.C. or PLC.

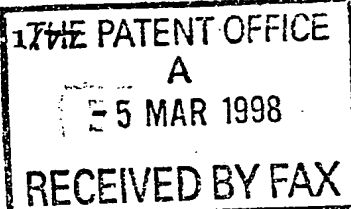
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Dated 21 August 2007

Patents Form 1



Patents Act 1977  
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The  
Patent  
Office

05MAR98 E343134-1 D02716  
P0177700 25.00 - 9804600.6

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(See the notes on the back of this form. You can also get an explanatory leaflet from the Patent Office to help you fill in this form)

The Patent Office

Cardiff Road  
Newport  
Gwent NP9 1RH

1. Your reference

PAT 98005 GB

2. Patent

**9804600.6**

**05 MAR 1998**

(The Patent Office will fill in this part)

3. Full name, address and postcode of the or of each applicant (underline all surnames)

NOKIA MOBILE PHONES LIMITED  
KEILALAHDENTIE 4  
02150 ESPOO  
FINLAND

Patents ADP number (if you know it)

If the applicant is a corporate body, give the country/state of its incorporation

5911995004

4. Title of the invention

A CDMA MODULATOR/DEMODULATOR

5. Name of your agent (if you have one)

"Address for service" in the United Kingdom to which all correspondence should be sent (including the postcode)

MRS HELEN LOUISE HAWS  
NOKIA MOBILE PHONES  
PATENT DEPARTMENT  
ST GEORGES COURT  
ST GEORGES ROAD  
CAMBERLEY  
SURREY GU15 3QZ UK

Patents ADP number (if you know it)

6945539001

6. If you are declaring priority from one or more earlier patent applications, give the country and the date of filing of the or of each of these earlier applications and (if you know it) the or each application number

Country

Priority application number  
(if you know it)

Date of filing  
(day / month / year)

7. If this application is divided or otherwise derived from an earlier UK application, give the number and the filing date of the earlier application

Number of earlier application

Date of filing  
(day / month / year)

8. Is a statement of inventorship and of right to grant of a patent required in support of this request? (Answer Yes if:

- a) any applicant named in part 3 is not an inventor, or
- b) there is an inventor who is not named as an applicant, or
- c) any named applicant is a corporate body.

See note (d))

YES

I certify this to be a true copy.

G D Court.

Acting for Comptroller

Patents Form 1/77

P. 002

FAX: 00 44 1276 677720

NMP PATENTS UK

5-MAR. 98 (THU) 16:09

Job-986

R-399

P. 02

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Continuation sheets of this form

Description 28

Claim(s)

Abstract

Drawing(s)

10. If you are also filing any of the following, state how many against each item.

Priority documents

Translations of priority documents

Statement of inventorship and right to grant of a patent (Patents Form 7/77)

Request for preliminary examination and search (Patents Form 9/77)

Request for substantive examination (Patents Form 10/77)

Any other documents (please specify)

11. I/We request the grant of a patent on the basis of this application.

Signature

*H I Haws*

Date

5/3/98

H I HAWS - AGENT FOR THE APPLICANT

12. Name and daytime telephone number of person to contact in the United Kingdom

Miss K Jeffery - 01276 419538

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Patents Form 1/77

## ■ CDMA Patent

The process at the transmitter of CDMA is roughly

BitStream → Frame Builder → Gold Code Encoding → Modulator [Using LookUp Table]  
D/A → Filter [Switched Capacitor, Clock Frequency] → Filter [Resistors, Capacitors] → Transmitter

The corresponding process for CDMA receiver is

Complex Number Seq → CDMA Process → Channel Estimation → Demodulation [Using Viterbi] → BitStream

These processes are in fact similar to GSM so we could build a dual mode phone. We describe the GSM processes

The transmitter processes are listed below

BitStream → Frame Builder → Modulator [Using LookUp Table]  
D/A → Filter [Switched Capacitor, Clock Frequency] → Filter [Resistors, Capacitors] → Transmitter

At the receiver we have the reverse process

Complex Number Seq → Multiplying incoming stream by  $i^n$  → Bit Synchronisation → Channel Estimation → Demodulation [Using Viterbi] → BitStream

### Note

- 1) The mode dependent processes are highlighted.
- 2) The same filter can be used in both the cases if the clock frequencies are selected with care.
- 3) The timing issues, and frequency dependent items like antennas have been ignored in this paper.
- 4) In this paper we use many different pulses depending on the bandwidth, chip rate, required and we trade off the non constant amplitude. This is the key innovation (to get higher bit rates).  
The selection of bitrate is just a matter of engineering design involving BER, Power Amplifier efficiency and spectrum utilisation. We also point out that although in this paper we have designed a modulator from first principles, we would in practice use the current Nokia Mobile Phones lookup table architecture [4T or 6T] obtaining the correct cyclostationary spectrum in each mode by the appropriate filter [Analog]. The Switched Capacitor will in practice do most of the shaping common to both. This again is subject to design.
- 5) The key innovative step is the nature of the function used to transform the gold code into the sequence used in the correlator by the function below. It is this function that we wish to protect in the patent. The transformation need only occur once, and the results stored, as the mobile can pre calculate the sequence  $d_i$  for  $i = 0, 1, \dots, N-1$  given the code  $\{c_0, c_1, \dots, c_{N-1}\}$  when the code is assigned.

$$y_i = (-1)^i \text{ for } i = 0, 1, 2, \dots, N-1$$

Given code  $\{c_0, c_1, \dots, c_{N-1}\}$  where  $N$  is the number of elements in the sequence.

$$a_i = 1 \text{ if } c_i = 1; \text{ for } i = 0, \dots, N-1$$

$$a_i = -1 \text{ if } c_i = 0; \text{ for } i = 0, \dots, N-1$$

$$b_0 = a_0$$

$$b_i = b_{i-1} + a_i \text{ for } i = 1, 2, \dots, N-1$$

$$d_i = y_i \cdot b_i \text{ for } i = 0, 1, 2, \dots, N-1 \text{ and}$$

$$i = \sqrt{-1}$$

The function is behaviour is specified in the variable  $x_4$  in the CDMA Coarse Synchroniser. We also stress that while the bitstream is differentially encoded and the substitution used is  $\{0 \rightarrow 1, 1 \rightarrow -1\}$  the substitution elsewhere used for the gold codes is  $\{0 \rightarrow -1, 1 \rightarrow 1\}$

6) The demodulation in the CDMA mode does not need a Viterbi algorithm as is required by the GSM mode. Thus, most of the work is done by the gold code during correlation.

7) There is nothing special about the Gold Code used in the example. Any gold like code will do as long as it possesses the usual properties of a gold code.

8) It is not necessary to have a long training sequence as in GSM, because the code is longer & M-sequence, which is in fact the GSM training seq.

The function is behaviour is specified in the variable x4 in the CDMA Coarse Synchroniser. We also stress that while the bitstream is differentially encoded and the substitution used is  $\{0 \rightarrow 1, 1 \rightarrow -1\}$  the substitution elsewhere used for the gold codes is  $\{0 \rightarrow -1, 1 \rightarrow 1\}$

6) The demodulation in the CDMA mode does not need a Viterbi algorithm as is required by the GSM mode. Thus, most of the work is done by the gold code during correlation.

7) There is nothing special about the Gold Code used in the example. Any gold like code will do as long as it possesses the usual properties of a gold code.

8) It is not necessary to have a long training sequence as in GSM, because all the work is done by the code.

## ■ Dual Mode CDMA and GSM Operation

In this work we demonstrate how to build a dual mode phone using as much as possible the same hardware for both the systems.

Given the following process for GSM Transmitter

Stream → Frame Builder → Modulator [Using LookUp Table]  
D/A → Filter[ Switched Capacitor,Clock Frequency] → Filter[ Resistors,Capacitors] → Transmitter

At the receiver we have the reverse process

Complex Number Seq → Channel Estimation → Demodulation [Using Viterbi] → BitStream

The corresponding process at the CDMA transmitter is roughly

BitStream → Frame Builder → GoldCode Encoding → Modulator [Using LookUp Table]  
D/A → Filter[ Switched Capacitor,Clock Frequency] → Filter[ Resistors,Capacitors] → Transmitter

The corresponding process for CDMA receiver is

Complex Number Seq → CDMA Process → Channel Estimation → Demodulation [Using Binary Decision] → BitStream

### Note

1)The mode dependent process are highlighted.

2) The same filter can be used in both the cases if the clock frequencies are selected with care.

3)The timing issues, and frequency dependent items like antennas have been ignored in this paper.

4)In this paper we use many different pulses depending on the bandwidth, chip rate, required and we trade off the non constant amplitude. This is the key innovation (to get higher bit rates).

The selection of bitrate is just a matter of engineering design involving BER, Power Amplifier efficiency and spectrum utilisation. We also point out that although in this paper we have designed a modulator from first principles, we would in practice use the current Nokia Mobile Phones lookup table architecture [ based on a pulse width of 4T ] or an architecture based on 6T pulse width obtaining the correct cyclostationary spectrum in each mode by the appropriate filter [Analog]. The Switched Capacitor will in practice do most of the spectral shaping common to both modes. This again is standard to design

5)The key innovative step is the nature of the function used to transform the gold code into the sequence used in the correlator by the function below. It is this function that we wish to protect in the patent. The transformation need only occur once, and the results stored, as the mobile can pre calculate the sequence  $d_i$  for  $i = 0, 1, \dots, N-1$  given the code  $\{c_0, c_1, \dots, c_{N-1}\}$  when the code is assigned.

$$y_i = (-1)^i \text{ for } i = 0, 1, 2, \dots, N-1$$

Given code  $\{c_0, c_1, \dots, c_{N-1}\}$  where  $N$  is the number of elements in the sequence.

$$a_i = 1 \text{ if } c_i = 1; \text{ for } i = 0, \dots, N-1$$

$$a_i = -1 \text{ if } c_i = 0; \text{ for } i = 0, \dots, N-1$$

$$b_0 = a_0;$$

$$b_i = b_{i-1} + a_i \text{ for } i = 1, 2, \dots, N-1;$$

$$d_i = y_i \cdot b_i \text{ for } i = 0, 1, 2, \dots, N-1 \text{ and}$$

$$i = \sqrt{-1}$$



```

In[1]:= Needs["RingFunctionsDiffEncoded`"]
In[2]:= Names["RingFunctionsDiffEncoded`*"]
Out[2]= {AllGoldSequences, AutocorrelationSequence,
AutomorphismSigma, CrosscorrelationSequence, CyclicMultiplativeGroup,
DropLeadingZeros, GoldSequence, GraeffeMethod, InitialConditions, MinimumPoly,
ModuloMultiplication, PolynomialMultiplication, PossibleDivisors, RingDivision,
RingPower, SequenceGenerator, SpecifiedGoldSequences, TraceRepresentation,
TupleRepresentation, TwoAdicExpansion, UnitsRing, ZeroPad, ZeroSequences,  $\tau$ ,  $\sigma$ }

In[3]:= Needs["LaurentFunctions`"]
RuleDelayed::rhs : Pattern t_ appears on the right-hand side of rule
PhaseAngle[L_][t_] > (PhaseAngle[L][t_] = Module[{x1, x2, x3, x4, x5, x6}, <<1>>]).

In[4]:= Needs["LaurentNotationTest`"]
Needs::nocont : Context LaurentNotationTest` was not created when Needs was evaluated.

Information on the functions used can be obtained using help.

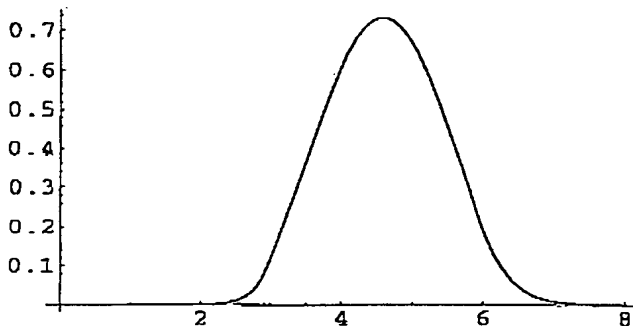
In[5]:= Names["LaurentFunctions`*"]
Out[5]= {AKN, AlphaKI, ANKInitialStateSetup, BT, FiltPulse, h, hFiltered, InitialState, J,
LaurentC, LaurentLK, LaurentS, M, ModulatingPulse, ModulationIndex, Modulator,
NumberOfCurves, PhaseAngle, PhaseAngleFast, Receiver, ReceiverProper, S,
SamplingInterval, StartingQuadrant, SyncSample, T, C,  $\Phi$ ,  $\psi$ }

In[6]:= T :=  $\frac{3}{812500}$ 
BT := 0.3

In[8]:= ModulationIndex :=  $\frac{1}{2}$ 

In[9]:= << ModulatorData.m;
In[10]:= << OptimalPulseShapes.m;
In[11]:= Plot[OptPulse[L][0][t], {t, 0, 8}]

```



Out[11]= - Graphics -





The number of samples per chip is equal to

$$\frac{3}{26000000} / T$$

$$\frac{1}{32}$$

[illegible]

```
In[95]:= ModOutputOpt = Modulator[L][(Goldseq1st // Last) /. {0 -> -1},
    NumberOfCurves -> 2, ModulatingPulse -> OptPulseScaled, SamplingInterval -> T/4];
```

[illegible]

This is a primitive decoder built to study the autocorrelation. This will help in decoding

```

In[51]:= PrimitiveCDMAReceiver[ModOutput_, GoldSeq_, Sample_, OverSampling_] :=
Module[{x1, x2State, x3Update, x4, x5State, x6},
  x1 = Partition[ModOutput, OverSampling] // Transpose // #[[Sample]]&;
  x2State = GoldSeq /. {0 -> -1};
  x5State = Table[(-1)^Mod[i, 2], {i, 0, Length[x2State] - 1}];
  x3Update := Module[{},
    x2State = RotateRight[x2State]; x4 = FoldList[Plus, 0, x2State] // Rest // I^#&;
    x5State = RotateRight[x5State]; x1 (x5State x4) // Apply[Plus, #]&;
    Table[x3Update, {1, 1, Length[GoldSeq]}]]

```

```

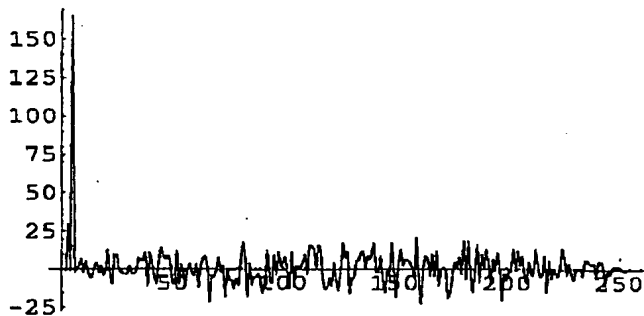
In[52]:= Tom = PrimitiveCDMAReceiver[ModOutputPlusOne, (GoldSeqList // Last), 1, 4];

```

```

In[53]:= ListPlot[Tom // Im, PlotJoined -> True, PlotRange -> All]

```



```

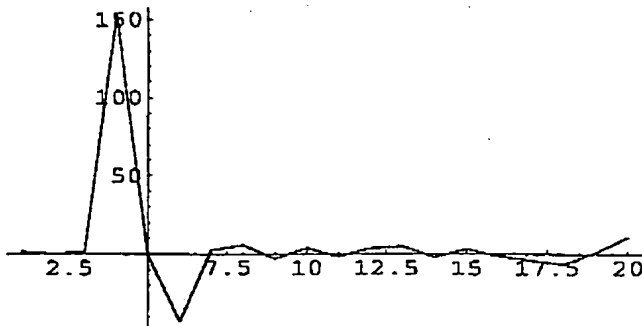
Out[53]= - Graphics -

```

```

In[54]:= ListPlot[Tom // Re // Take[#, 20]&, PlotJoined -> True, PlotRange -> All]

```



```

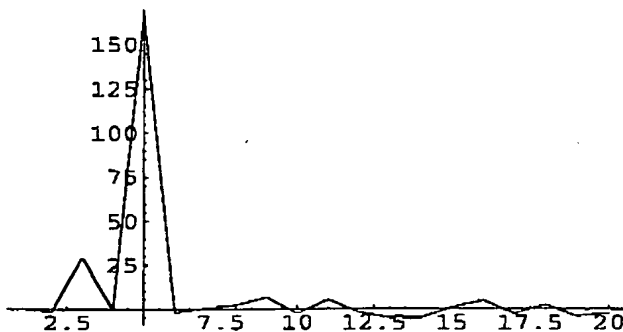
Out[54]= - Graphics -

```

```

In[55]:= ListPlot[Tom // Im // Take[#, 20]&, PlotJoined -> True, PlotRange -> All]

```



```

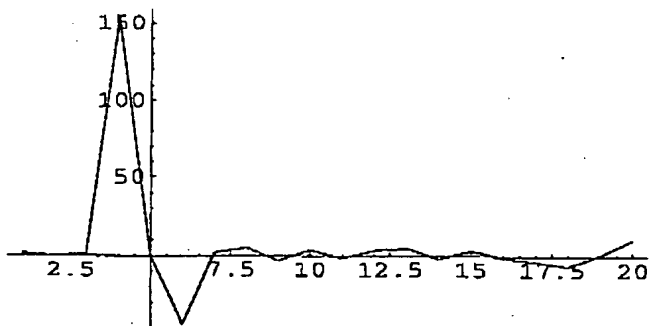
Out[55]= - Graphics -

```

```
In[56]:= Take[Tom, 10]
```

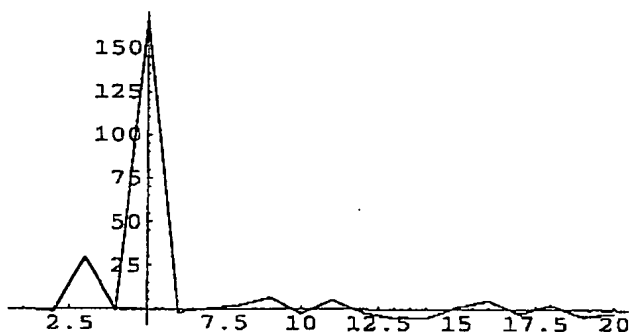
```
Out[56]:= {1.75501 + 0.329995 I, 0.0343421 - 1.58366 I, 1.39349 + 29.6902 I, 153.017 - 0.957068 I,
-0.718377 + 165.563 I, -43.6424 - 2.04777 I, 2.21031 + 0.256636 I, 5.392 + 2.18642 I,
-3.13017 + 6.54167 I, 3.61193 - 2.5452 I}
```

```
In[57]:= ListPlot[Tom // Re // Take[#, 20]&, PlotJoined -> True, PlotRange -> All]
```



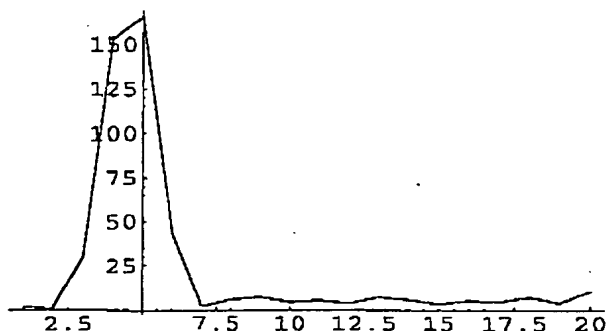
```
Out[57]:= - Graphics -
```

```
In[58]:= ListPlot[Tom // Im // Take[#, 20]&, PlotJoined -> True, PlotRange -> All]
```



```
Out[58]:= - Graphics -
```

```
In[59]:= ListPlot[Tom // Abs // Take[#, 20]&, PlotJoined -> True, PlotRange -> All]
```



```
Out[59]:= - Graphics -
```

We try a less favourable sequence

```
In[60]:= ModOutputPlusOne3 = Modulator[L][CDMAEncode[{1}, Goldseqlist // #[[3]]&,
NumberOfCurves -> 2, ModulatingPulse -> OptPulseScaled, SamplingInterval -> T/4];
```

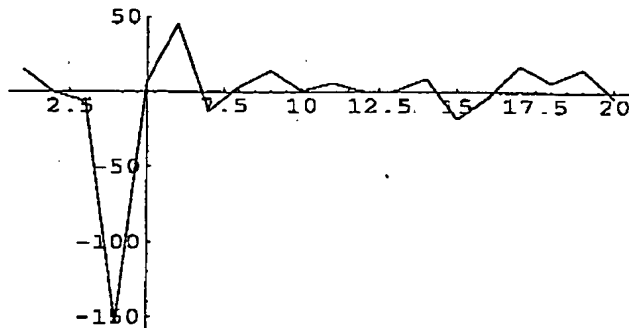
```

In[68]:= Tom4 = PrimitiveCDMAReceiver[ModOutputPlusOne3, (Goldseq1st // #[[3]]&), 1, 4];
In[69]:= Take[Tom4, 10]

Out[69]= {14.7675 - 4.24542 I, -0.301874 + 8.61808 I, -5.92939 + 29.6038 I, -154.61 - 10.4754 I,
6.64138 + 166.152 I, 45.2887 - 7.16233 I, -12.809 + 2.47796 I, 3.45948 - 19.7653 I,
14.2097 + 6.33624 I, 0.585872 - 6.10244 I}

In[70]:= ListPlot[Tom4 // Re // Take[#, 20]&, PlotJoined -> True, PlotRange -> All]

```

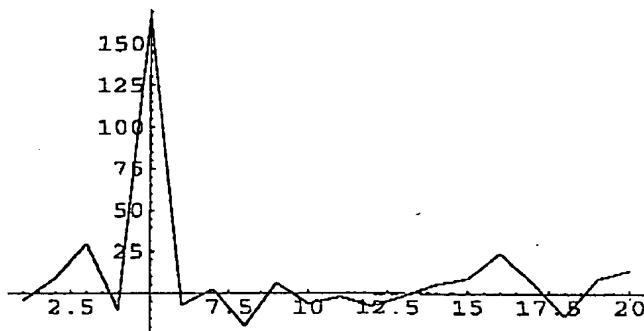


```
Out[70]= - Graphics -
```

```

In[71]:= ListPlot[Tom4 // Im // Take[#, 20]&, PlotJoined -> True, PlotRange -> All]

```



```
Out[71]= - Graphics -
```

### ■ CDMA Decoding of Several Symbols with Training sequence Receiver

First we generate the modulator output. For simplicity we will use a very short training sequence. Let the training sequence be  $\{1, -1, -1, 1\}$ . Let the guard sequences be  $\{1, 1, 1\}$ . Let the data symbols be generated by a random

```

In[72]:= data1 = {1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 0, 1, 1, 1, 0, 1, 1, 0, 1, 0};
In[73]:= data2 = {0, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0, 1, 1};
In[74]:= guard = {1, 1, 1}
Out[74]= {1, 1, 1}

In[75]:= training = {0, 0, 1, 0, 0, 1, 0, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1, 1};

```

The GSM Training sequence is  $\{0, 0, 1, 0, 0, 1, 0, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1, 1\}$ . In fact any short m-sequence can be used to characterise the output.



only at this point that we differentially encode using the gsm scheme

```
In[76]:= frame = Join[guard, data1, training, data2, guard]

General::spell1 :
Possible spelling error: new symbol name "frame" is similar to existing symbol "Frame".

Out[76]:= {1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 0, 1, 1, 1, 0, 1, 1, 0, 1, 0, 0, 0, 1, 0,
0, 1, 0, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1, 1, 0, 0, 1, 0, 1,
1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 1, 1}

In[81]:= GSMDiffEncodedFrame[frame_] := Module[{x1, x2, x3}, x1 = Partition[frame, 2, 1];
x2 = Map[Mod[#[[1]] + #[[2]], 2]&, x1] // Join[{frame[[1]]}, #]&;
x3 = x2 /. {0 -> 1, 1 -> -1}

In[83]:= frameEncoded = GSMDiffEncodedFrame[frame]

Out[83]:= {-1, 1, 1, 1, 1, 1, 1, 1, 1, -1, -1, -1, -1, -1, -1, 1, 1, -1, -1, 1, -1, -1, -1, 1, 1,
-1, -1, 1, -1, -1, -1, 1, 1, -1, 1, 1, 1, -1, -1, 1, 1, -1, -1, 1, -1, -1, -1,
1, 1, -1, 1, -1, -1, -1, 1, -1, -1, -1, -1, 1, -1, -1, 1, -1, -1, 1, 1, 1, 1}

In[84]:= ModOutputFrame = Modulator[L][CDMAEncode[frameEncoded, Goldseqlist // Last],
NumberOfCurves -> 2, ModulatingPulse -> OptPulseScaled, SamplingInterval -> T/4];

In[85]:= Save["ModOutputFrameEncodedGSMLike.m", ModOutputFrame]

<< ModOutputFrame.m;

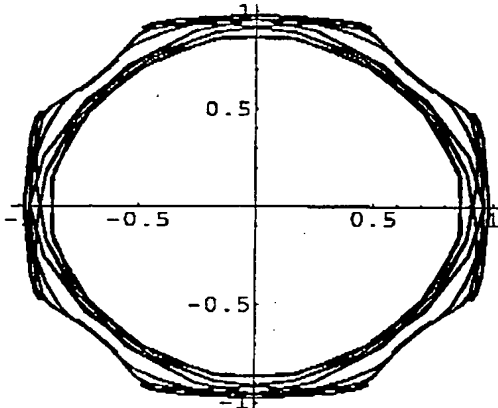
In[86]:= Length[ModOutputFrame]

Out[86]:= 73440

In[87]:= TestData = Take[ModOutputFrame, {50, 7300}];

General::spell1 : Possible spelling error: new symbol name "TestData" is similar
to existing symbol "TextData".

In[88]:= TestData // {Re[#], Im[#]}& // Transpose //
ListPlot[#, PlotJoined -> True, PlotRange -> All, AspectRatio -> 1]&;
```



In the PrimitiveCDMA receiver we need to specify the sample. In the CDMA synchroniser we discover the sample.

```

In[89]:= CDMACoarseSynchroniser[ModOutput_, GoldSeq_, Threshold_, OverSampling_] :=
Module[{x1, x2, x3, x4, x5State,
  x6Count, x6MaxCorr, seq, x7Update, x8, x9, x10, x11, x12, x13},
  x1 = ModOutput;
  x2 = GoldSeq /. 0 -> -1;
  x3 = Table[(-1)^Mod[i, 2], {i, 0, Length[x2] - 1}];
  x4 = FoldList[Plus, 0, x2] // Rest // I^#& // x3 #&;
  x5State = Take[x1, (Length[x2] + 1) OverSampling];
  x6Count = 1;
  x6MaxCorr = 0;
  seq = Drop[x1, OverSampling (Length[x2] + 1)];
  x7Update := Module[{}];
  x8 = Partition[x5State, OverSampling] // Transpose;
  x9 = Map[{Drop[#, -1] . x4, Drop[#, 1] . x4}&, x8];
  x10 = Map[{Abs[Im[#[[1]]]],
    Abs[Re[#[[1]]]], Abs[Re[#[[2]]]], Abs[Im[#[[2]]]] }&, x9];
  x11 = If[Max[x10] > Threshold, Throw[{x10, x6Count, True}],
    {x6Count, x10, x6MaxCorr = Max[x6MaxCorr, x10], False}];
  x6Count = x6Count + 1;
  x5State = Join[Drop[x5State, OverSampling], Take[seq, OverSampling]];
  seq = Drop[seq, OverSampling];
  x11 ];
  x12 = Catch[Table[x7Update, {i, 1, Length[seq] / OverSampling}]];
  x13 = If[Last[x12] === True,
    CDMAFineSynchroniser[ModOutput, x12[[2]], x4, OverSampling],
    {"Failed to Coarse Synchronise", False}]]

In[90]:= CDMAFineSynchroniser[ModOutput_,
  ThresholdCorrelatioCount_, CorrelatingSeq_, OverSampling_] :=
Module[{x1, x2, x3, x4, x5, x6, seq, x7Update, x8First,
  x8Second, x9First, x9Second, x10First, x10Second, x11, x12, x13, x14},
  x1 = If[ThresholdCorrelatioCount > 3,
    ThresholdCorrelatioCount - 3, ThresholdCorrelatioCount];
  x2 = CorrelatingSeq;
  x3 = Drop[ModOutput, x1 OverSampling];
  x4 = CDMAPositionFinder[x3, x2, OverSampling];
  x5 = CDMAPositionFinder[Drop[x3, Length[x2] OverSampling], x2, OverSampling];
  x6 = CDMAPositionFinder[Drop[x3, 2 Length[x2] OverSampling], x2, OverSampling];
  x7 = PositionAverager[{x4[[1]], x5[[1]], x6[[1]]}];
  x8First = Drop[x3, (x7[[1]] - 1) OverSampling] //
    Partition[#, OverSampling]& // Transpose // #[[x7[[2]]]]&;
  x8Second = Drop[x3, x7[[1]] OverSampling] //
    Partition[#, OverSampling]& // Transpose // #[[x7[[2]]]]&;
  x9First = x8First // Partition[#, Length[x2]]&;
  x10First = Map[Function[x, x.x2], x9First];
  x9Second = x8Second // Partition[#, Length[x2]]&;
  x10Second = Map[Function[x, x.x2], x9Second];
  DeModulator[{x10First, x10Second}] ]

In[91]:= CDMAPositionFinder[ModOutput_, CorrelatingSeq_, OverSampling_] :=
Module[{x5State, x6Count, seq, x7Update, x8, x9, x10, x11, x12, x13},
  x5State = Take[ModOutput, (Length[CorrelatingSeq] + 1) OverSampling];
  x6Count = 1;
  seq = Drop[ModOutput, OverSampling (Length[CorrelatingSeq] + 1)];
  x7Update := Module[{}];
  x8 = Partition[x5State, OverSampling] // Transpose;
  x9 = Map[{Drop[#, -1] . CorrelatingSeq, Drop[#, 1] . CorrelatingSeq}&, x8];
  x6Count = x6Count + 1;
  x5State = Join[Drop[x5State, OverSampling], Take[seq, OverSampling]];
  seq = Drop[seq, OverSampling];
  x9 ];
  x10 = Table[x7Update, {i, 1, 10}];
  x11 = MapIndexed[Max[{Abs[Re[#]], Abs[Im[#]]}]&, x10, {3}];
  x12 = MapIndexed[Apply[Plus, #]&, x11, {2}];
  x13 = Position[x12, Max[x12]] ]

```

We need to define a position averager. We for now just take the first element

```

In[98]:= PositionAverager[PositionList_] := First[PositionList]

```

```

In[101]:= Tom4 =
  CDMACoarseSynchroniser[TestData // Drop[#, 250 4]&, Goldseqlist // Last, 100, 4]

Out[101]= DeModulator[{{-157.056 + 0.691379 I, -0.691379 - 157.056 I,
  157.056 - 0.691379 I, 0.691379 + 157.056 I, -157.056 + 0.691379 I,
  {-169.734 - 0.510132 I, 0.510132 - 169.734 I, 169.734 + 0.510132 I,
  -0.510132 + 169.734 I, -169.734 - 0.510132 I}}}]]

In[102]:= Tom5 = CDMACoarseSynchroniser[ModOutputFrame // Drop[#, 250 4]&,
  Goldseqlist // Last, 100, 4]

Out[102]= DeModulator[{{1.02745 + 156.682 I, -157.056 + 0.691379 I, -0.691379 - 157.056 I,
  157.056 - 0.691379 I, 0.691379 + 157.056 I, -157.056 + 0.691379 I,
  -0.691379 - 157.056 I, 157.25 - 0.929033 I, 19.1272 - 6.93509 I, -7.37139 - 18.8305 I,
  -18.8305 + 7.37139 I, 7.37139 + 18.8305 I, 18.8305 - 7.37139 I, -7.57128 - 19.0518 I,
  -1.02745 - 156.682 I, 157.25 - 0.929033 I, 19.1272 - 6.93509 I, -7.57128 - 19.0518 I,
  -1.2651 - 156.875 I, -6.93509 - 19.1272 I, -18.8305 + 7.37139 I, 7.57128 + 19.0518 I,
  1.02745 + 156.682 I, -157.25 + 0.929033 I, -19.1272 + 6.93509 I, 7.57128 + 19.0518 I,
  1.2651 + 156.875 I, 6.93509 + 19.1272 I, 18.8305 - 7.37139 I, -7.57128 - 19.0518 I,
  -1.02745 - 156.682 I, 157.25 - 0.929033 I, 19.3484 - 7.13498 I, 156.682 - 1.02745 I,
  0.691379 + 157.056 I, -157.25 + 0.929033 I, -19.1272 + 6.93509 I, 7.57128 + 19.0518 I,
  1.02745 + 156.682 I, -157.25 + 0.929033 I, -19.1272 + 6.93509 I, 7.57128 + 19.0518 I,
  1.2651 + 156.875 I, 6.93509 + 19.1272 I, 18.8305 - 7.37139 I, -7.57128 - 19.0518 I,
  -1.02745 - 156.682 I, 157.25 - 0.929033 I, 19.3484 - 7.13498 I, 156.875 - 1.2651 I,
  19.1272 - 6.93509 I, -7.37139 - 18.8305 I, -19.0518 + 7.57128 I, -156.875 + 1.2651 I,
  -19.1272 + 6.93509 I, 7.37139 + 18.8305 I, 18.8305 - 7.37139 I, -7.57128 - 19.0518 I,
  -1.2651 - 156.875 I, -6.93509 - 19.1272 I, -19.0518 + 7.57128 I,
  -156.875 + 1.2651 I, -19.1272 + 6.93509 I, 7.37139 + 18.8305 I,
  19.0518 - 7.57128 I, 156.875 - 1.2651 I, 19.3484 - 7.13498 I, 156.682 - 1.02745 I,
  0.691379 + 157.056 I, -157.056 + 0.691379 I, {-0.478141 + 169.715 I,
  -169.734 - 0.510132 I, 0.510132 - 169.734 I, 169.734 + 0.510132 I,
  -0.510132 + 169.734 I, -169.734 - 0.510132 I, 0.510132 - 169.734 I,
  169.813 - 0.714545 I, 20.5326 - 6.8667 I, -6.8871 - 20.5026 I, -20.5026 + 6.8871 I,
  6.8871 + 20.5026 I, 20.5026 - 6.8871 I, -7.42576 - 21.3328 I, 0.478141 - 169.715 I,
  169.813 - 0.714545 I, 20.5326 - 6.8667 I, -7.42576 - 21.3328 I, -0.746535 - 169.794 I,
  -6.8667 - 20.5326 I, -20.5026 + 6.8871 I, 7.42576 + 21.3328 I, -0.478141 + 169.715 I,
  -169.813 + 0.714545 I, -20.5326 + 6.8667 I, 7.42576 + 21.3328 I, 0.746535 + 169.794 I,
  6.8667 + 20.5326 I, 20.5026 - 6.8871 I, -7.42576 - 21.3328 I, 0.478141 - 169.715 I,
  169.813 - 0.714545 I, 21.3629 - 7.40536 I, 169.715 + 0.478141 I,
  -0.510132 + 169.734 I, -169.813 + 0.714545 I, -20.5326 + 6.8667 I,
  7.42576 + 21.3328 I, -0.478141 + 169.715 I, -169.813 + 0.714545 I,
  -20.5326 + 6.8667 I, 7.42576 + 21.3328 I, 0.746535 + 169.794 I, 6.8667 + 20.5326 I,
  20.5026 - 6.8871 I, -7.42576 - 21.3328 I, 0.478141 - 169.715 I, 169.813 - 0.714545 I,
  21.3629 - 7.40536 I, 169.794 - 0.746535 I, 20.5326 - 6.8667 I, -6.8871 - 20.5026 I,
  -21.3328 + 7.42576 I, -169.794 + 0.746535 I, -20.5326 + 6.8667 I,
  20.5026 - 6.8871 I, -7.42576 - 21.3328 I, -0.746535 - 169.794 I, -6.8667 - 20.5326 I,
  -21.3328 + 7.42576 I, -169.794 + 0.746535 I, -20.5326 + 6.8667 I,
  6.8871 + 20.5026 I, 21.3328 - 7.42576 I, 169.794 - 0.746535 I, 21.3629 - 7.40536 I,
  169.715 + 0.478141 I, -0.510132 + 169.734 I, -169.734 - 0.510132 I}}}]]

In[105]:= SeqI = Table[I^(1) (-1)^(1 + 1), {1, 1, Length[Tom5][[1, 1]]}]

Out[105]= {I, 1, -I, -1, I, 1, -I, -1, I, 1, -I, -1, I, 1, -I, -1, I, 1, -I,
  -1, I, 1, -I, -1, I, 1, -I, -1, I, 1, -I, -1, I, 1, -I, -1, I,
  1, -I, -1, I, 1, -I, -1, I, 1, -I, -1, I, 1, -I, -1, I, 1, -I, -1, I, 1}

```

```
In[106]:= Tom5[[1, 1]] SeqI
```

```
Out[106]= {-156.682+1.02745 I, -157.056+0.691379 I, -157.056+0.691379 I,
-157.056+0.691379 I, -157.056+0.691379 I, -157.056+0.691379 I,
-157.056+0.691379 I, -157.25+0.929033 I, 6.93509+19.1272 I, -7.37139-18.8305 I,
7.37139+18.8305 I, -7.37139-18.8305 I, 7.37139+18.8305 I, -7.57128-19.0518 I,
-156.682+1.02745 I, -157.25+0.929033 I, 6.93509+19.1272 I, -7.57128-19.0518 I,
-156.875+1.2651 I, 6.93509+19.1272 I, -7.37139-18.8305 I, 7.57128+19.0518 I,
156.682-1.02745 I, 157.25-0.929033 I, -6.93509-19.1272 I, 7.57128+19.0518 I,
156.875-1.2651 I, -6.93509-19.1272 I, 7.37139+18.8305 I, -7.57128-19.0518 I,
-156.682+1.02745 I, -157.25+0.929033 I, 7.13498+19.3484 I, 156.682-1.02745 I,
157.056-0.691379 I, 157.25-0.929033 I, -6.93509-19.1272 I, 7.57128+19.0518 I,
156.682-1.02745 I, 157.25-0.929033 I, -6.93509-19.1272 I, 7.57128+19.0518 I,
156.875-1.2651 I, -6.93509-19.1272 I, 7.37139+18.8305 I, -7.57128-19.0518 I,
-156.682+1.02745 I, -157.25+0.929033 I, 7.13498+19.3484 I, 156.875-1.2651 I,
-6.93509-19.1272 I, 7.37139+18.8305 I, -7.57128-19.0518 I, -156.875+1.2651 I,
6.93509+19.1272 I, -7.37139-18.8305 I, 7.37139+18.8305 I, -7.57128-19.0518 I,
-156.875+1.2651 I, 6.93509+19.1272 I, -7.57128-19.0518 I, -156.875+1.2651 I,
6.93509+19.1272 I, -7.37139-18.8305 I, 7.57128+19.0518 I, 156.875-1.2651 I,
-7.13498-19.3484 I, -156.682+1.02745 I, -157.056+0.691379 I,
-157.056+0.691379 I}
```

```
In[114]:= demodframe = Sign[Re[Tom5[[1, 1]] SeqI]] /. {-1 -> 1, 1 -> 0}
```

```
Out[114]= {1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0, 0, 1, 0,
0, 1, 0, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1, 1, 0, 0, 1, 0,
1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 1}
```

The first and last bits have been lost in the processing

```
In[109]:= frame
```

```
Out[109]= {1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0, 0, 1,
0, 0, 1, 0, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1, 1, 0, 0, 1,
0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 1, 1}
```

```
In[122]:= Length[frame]
```

```
Out[122]= 72
```

```
In[115]:= truncatedframe = frame // Drop[#, 1]& // Drop[#, -1]&
```

```
Out[115]= {1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 1, 0, 0, 1, 0,
0, 1, 0, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1, 1, 0, 0, 1, 0,
1, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 0, 1, 1, 1, 1}
```

```
In[116]:= demodframe - truncatedframe
```

```
Out[116]= {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

## ■ References

1 "Binary Sequences with Gold-Like Correlation but Larger Linear Span" by  
Serdar Boztas and P. Vijay Kumar IEEE Trans. on Information Theory Vol 40 No 2,  
March 1984

```

BeginPackage["RingFunctionsDiffEncoded"]

RingDivision::usage = "RingDivision[ModuloRing][{Denominator,Numerator}] =
{Quotient,Remainder,Denominator,Flag} e.g If num = {1,0,0,0,0,0,0,0,1,
0,0,0,0,0,1},den = {1,0,0,1,0,1}, then RingDivision[2][{den,num}] = {{1,
0,0,1,0,1,1,0,0,0,1},{1,0,1,0,0},{1,0,0,1,0,1},True}. When the
algorithm encounters a zero divisor the flag is set to False. e.g RingDivision[
4][{{2,1,1,1},{1,0,0,1}}] = {{},{1,0,0,1},{2,1,1,1},False}"

DropLeadingZeros::usage =
"DropLeadingZeros[DenominatorSeq] drops the leading zeros in the sequence.
e.g DropLeadingZeros[{3,3,1}] = {3,3,1}. DropLeadingZeros[{0,0,0}] = {}"

PossibleDivisors::usage = "PossibleDivisors[ModuloRing][Order] gives the list of
possible divisors. Note the MS power of the poly is LHS. PossibleDivisors[4][
2] = {{1},{2},{3},{1,0},{1,1},{1,2},{1,3},{2,0},{2,1},{2,2},{2,3},{3,0},
{3,1},{3,2},{3,3},{1,0,0},{1,0,1},{1,0,2},{1,0,3},{1,1,0},{1,1,1},{1,
1,2},{1,1,3},{1,2,0},{1,2,1},{1,2,2},{1,2,3},{1,3,0},{1,3,1},{1,3,2},
{1,3,3},{2,0,0},{2,0,1},{2,0,2},{2,0,3},{2,1,0},{2,1,1},{2,1,2},{2,1,
3},{2,2,0},{2,2,1},{2,2,2},{2,2,3},{2,3,0},{2,3,1},{2,3,2},{2,3,3},
{3,0,0},{3,0,1},{3,0,2},{3,0,3},{3,1,0},{3,1,1},{3,1,2},{3,1,
3},{3,2,0},{3,2,1},{3,2,2},{3,2,3},{3,3,0},{3,3,1},{3,3,2},{3,3,3}}"

PolynomialMultiplication::usage = "PolynomialMultiplication[ModuloRing][
poly1,poly2] performs polynomial multiplication moduloRing e.g.
PolynomialMultiplication[4][{2,1},{2,1}] = {1}"

ModuloMultiplication::usage =
"ModuloMultiplication[ModuloRing][PrimitivePolynomial][poly1,poly2] e.
g ModuloMultiplication[4][{1,2,1,3}][{3,3,0},{3,3,0}] = {1,0}"

RingPower::usage = "RingPower[RingModulo][PrimitiveElement][Element] e.g RingPower[
4][{1,0,0,3,2,3}][{1,0,0}] = {{1,0,0},{1,0,0,0,0},{1,2,1,0},{2,1,1,
2,1},{1,0,2,0,1},{2,1,3,1,0},{3,3,1,0,1},{1,3,2,1,3},{2,2,0,3,
3},{1,1,2,2},{1,2,3,2,1},{3,3,1,1,2},{1,0,3,1,3},{3,2,1,1,
0},{1,0,0,3,2},{1,0},{1,0,0,0},{1,2,1},{1,2,1,0,0},{1,1,0,
1,2},{2,1,3,1},{1,3,3,0,2},{3,1,3,3,3},{3,2,2,1,1},{2,0,1,
3,2},{1,1,2,2,0},{2,3,3,3,1},{3,1,0,0,3},{3,2,1,1},{2,1,0,2,3},{1}}"

CyclicMultiplativeGroup::usage = "CyclicMultiplativeGroup[ModuloRing][
PrimitivePolynomial] e.g. CyclicMultiplativeGroup[4][{1,2,1,3}] = {{1},{1,
0},{1,0,0},{2,3,1},{3,3,2},{1,3,3},{1,2,1}} and CyclicMultiplativeGroup[
4][{1,0,0,3,2,3}] = {{1},{1,0,0},{1,0,0,0,0},{1,2,1,0},{2,1,1,2,1},
{1,0,2,0,1},{2,1,3,1,0},{3,3,1,0,1},{1,3,2,1,3},{2,2,0,3,3},{1,1,2,
2},{1,2,3,2,1},{3,3,1,1,2},{1,0,3,1,3},{3,2,1,1,0},{1,0,0,3,2},
{1,0},{1,0,0,0},{1,2,1},{1,2,1,0,0},{1,1,0,1,2},{2,1,3,1},
{1,3,3,0,2},{3,1,3,3,3},{3,2,2,1,1},{2,0,1,3,2},{1,1,2,2,
0},{2,3,3,3,1},{3,1,0,0,3},{3,2,1,1},{2,1,0,2,3}}"

ZeroSequences::usage =
"ZeroSequences[ModuloRing][Order] generates the zero ideal ZeroSequences[4][
3] = {{0,0,0},{0,0,2},{0,2,0},{0,2,2},{2,0,0},{2,0,2},{2,2,0},{2,2,2}}"

ZeroPad::usage = "ZeroPad[Order][Element] eg ZeroPad[3][{1}] = {0,0,1}"

```

```
T::usage = "T[Order_][σ_][x_] e.g. T[4][x3][{3, 0, 3}] = "
```

[illegible]

```

AllGoldSequences::usage = "AllGoldSequences[PrimitivePoly] e.g. AllGoldSequences[
(1,0,0,3,2,3)] = {{(0,0,0,0,0,0,0,0,0,1,0,0,0,1,0,1,1,0,1,1,1,0,
0,0,1,1,1,0,0,1,1),(1,0,0,0,0,1,0,0,1,1,1,1,0,1,1,0,0,1,0,1,1,
0,1,1,1,0,0,1,0,0,1),(0,1,0,0,0,0,1,0,0,0,0,1,1,1,0,0,0,1,0,0,0,
1,0,0,1,0,1,0,1,1,1,0),(1,1,0,0,0,1,1,0,1,0,1,0,1,1,1,1,1,0,1,
0,1,0,1,0,0,0,1,0,1,0,0),(0,0,1,0,0,1,0,1,1,1,0,1,1,0,1,0,1,0,
1,0,0,0,1,1,0,1,0,0,1,1,1),(1,0,1,0,0,0,0,1,0,1,1,0,1,0,0,1,0,
1,0,0,0,0,0,0,0,1,1,1,0,1),(0,1,1,0,0,1,1,1,1,0,0,0,0,0,1,1,
0,1,0,1,0,0,1,0,1,1,1,1,0,1,0),(1,1,1,0,0,0,0,1,0,0,1,1,0,0,0,
0,1,0,1,1,0,0,0,1,1,0,0,0,0,0,0),(0,0,0,1,0,0,1,0,1,0,0,0,1,0,
1,0,0,0,1,1,0,1,0,1,0,0,1,1,0,0,1),(1,0,0,1,0,1,1,0,0,0,0,1,1,
0,0,1,1,1,0,1,0,1,1,0,0,1,0,0,0,1,1),(0,1,0,1,0,0,0,0,0,1,1,0,1,
0,0,1,1,1,1,0,0,0,1,0,0,1,0,0,0,1,0,0),(1,1,0,1,0,1,0,0,0,1,1,
0,0,0,0,0,0,0,0,1,1,1,1,1,1,1,0),(0,0,1,1,0,1,1,1,1,0,0,0,
0,1,0,1,0,1,0,0,1,0,1,1,1,0,1,0,0,1,1,0,1),(1,0,1,1,0,0,1,1,1,
0,1,0,0,1,1,0,0,1,1,0,0,1,1,0,1,1,1,1,0,1,1,1),(0,1,1,1,0,1,0,0,1,
0,1,0,0,1,1,0,0,1,1,0,1,1,1,1,0,0,1,0,0,0,0),(1,1,1,1,1,0,0,0,
1,1,1,1,1,1,1,1,1,0,0,0,1,1,1,1,0,0,0,1,0,1,0,1,0),(0,0,0,0,1,0,
0,1,0,0,1,0,0,0,1,0,0,1,1,1,1,1,1,0,0,0,0,0,0,1,1,0),(1,0,0,0,1,
1,0,1,1,0,0,0,1,0,0,0,1,1,0,0,1,0,1,1,1,0,1,0,1,1,0,0),(0,1,0,0,
1,0,1,1,0,1,1,1,1,0,1,1,1,0,0,0,1,1,1,1,1,0,1,1,0,1,1),(1,1,0,
0,1,1,1,1,1,1,1,0,0,0,0,0,1,1,1,1,1,0,1,1,0,1,1,0,1,1),(1,1,0,
0,1,1,1,1,1,1,1,0,0,0,0,0,0,1,1,1,1,1,0,1,1,0,1,1,0,1,1),(1,1,0,
0,1,1,1,1,1,1,1,0,0,0,0,0,0,0,1,1,1,1,1,0,1,1,0,1,1,0,1,1),(0,0,
0,1,1,1,1,1,1,1,0,0,0,0,0,0,0,1,1,0,1,1,0,1,1,0,0,0,1),(0,0,
1,0,1,1,0,0,1,0,1,1,1,1,0,1,0,1,1,0,0,1,0,1,1,0,0,1,0),(1,
1,0,1,0,0,0,0,0,0,0,1,1,1,0,1,0,0,0,0,1,1,0,1,1,0,1,0,0,0),
(0,1,1,0,1,1,1,0,1,1,1,0,0,1,0,0,1,0,0,1,0,1,0,0,0,0,1,1,1,
1),(1,1,1,0,1,0,1,0,0,1,0,1,0,1,1,1,0,1,1,1,0,1,1,1,0,1,1,0,1,
0,1),(0,0,1,1,0,1,1,1,1,1,0,1,1,0,1,1,1,1,1,0,0,1,1,1,0,1,
1,0,0),(1,0,0,1,1,1,1,1,0,1,0,1,1,1,1,1,0,0,0,0,1,0,0,0,0,1,0,1,
0,1,1,0),(0,1,0,1,1,0,0,1,1,0,1,1,0,1,0,0,0,0,0,0,0,0,1,0,0,1,
1,0,0,0,1),(1,1,0,1,1,1,0,1,0,0,0,0,0,1,1,1,1,1,1,0,0,0,0,1,0,
0,0,1,0,1,1),(0,0,1,1,1,1,1,0,0,1,1,1,0,0,1,0,1,1,1,0,1,1,1,0,1,
0,0,0,0,1,1,1,0,0,0),(1,0,1,1,1,0,1,0,1,1,0,0,0,0,0,1,
0,0,0,0,1,0,1,0,1,0,0,0,0,1,0),(0,1,1,1,1,1,0,0,0,0,1,
0,1,0,1,1,0,0,0,1,1,0,0,1,1,1,0,0,1,0,1),(1,1,1,1,1,1,0,
0,0,1,0,0,1,1,0,0,0,1,1,1,1,0,1,0,1,0,1,1,1,1,1),(1,
0,0,0,0,1,0,0,1,0,1,1,0,0,0,1,1,1,1,1,0,0,0,1,1,0,1,1,0,1,0}}"
```

```

CrosscorrelationSequence::usage = "CrosscorrelationSequence[{GoldSeq1,GoldSeq2}]
e.g. seqlist = SpecifiedGoldSequences[{1,3,2,0,1,1,1,1,1,1,1,1}][{1,2,3,
4,5,6,7,8,Last}];Outer[CrosscorrelationSequence[{#1,#2}]]//Union &,
seqlist,seqlist,1] = {{{{-33,-1,31,1023},{-65,-33,-1,31,63},{-65,-33,-1,
31,63},{-65,-33,-1,31,63},{-65,-33,-1,31,63},{-65,-33,-1,31,63},{-65,-33,-1,
31,63},{-65,-33,-1,31,63},{-33,31},{-65,-33,-1,
31,63},{-33,-1,31,1023},{-65,-33,-1,31,63},{-65,-33,-1,31,63},{-65,-33,
-1,31,63},{-65,-33,-1,31,63},{-65,-33,-1,31,63},{-65,-33,-1,31,63},{-65,
-33,-1,31,63},{-33,31},{-65,-33,-1,31,63},{-65,-33,-1,31,63},{-65,-33,
-1,31,63,1023},{-33,-1,31},{-65,-33,-1,31,63},{-65,-33,-1,31,63},{-65,
-33,-1,31,63},{-65,-33,-1,31,63},{-65,-33,-1,31,63},{-33,31},{-65,-33,
-33,-1,31,63},{-65,-33,-1,31,63},{-65,-33,-1,31,63},{-65,-33,-1,31,
63},{-65,-33,-1,31,63},{-33,-1,31,1023},{-65,-33,-1,31,63},{-65,-33,-1,31,
63},{-65,-33,-1,31,63},{-33,31},{-65,-33,-1,31,63},{-65,-33,-1,31,63},
{-65,-33,-1,31,63,1023},{-33,-1,31},{-65,-33,-1,31,63},{-33,31},{-65,
-33,-1,31,63},{-65,-33,-1,31,63},{-65,-33,-1,31,63},{-65,-33,-1,31,
63},{-65,-33,-1,31,63},{-65,-33,-1,31,63},{-65,-33,-1,31,63},{-65,-33,-1,
31,63},{-65,-33,-1,31,63},{-65,-33,-1,31,63},{-33,-1,31},{-65,-33,-1,31,
63,1023},{-65,-33,-1,31,63},{-33,31},{-65,-33,-1,31,63},{-65,-33,-1,31,
63},{-65,-33,-1,31,63},{-65,-33,-1,31,63},{-65,-33,-1,31,63},{-33,
-1,31,1023},{-33,31},{-33,31},{-33,31},{-33,31},{-33,31},{-33,
31},{-33,31},{-33,31},{-33,31},{-33,31},{-1,1023}}}"
```

```
Begin["Private"]
```



```

RingDivision[ModuloRing_][{Denominator_, Numerator_}] :=
Module[{x1, x2, x3, x4, x5, x6, Qx, Nx},
Clear[x1];
x1[i_, j_] := Mod[i j, ModuloRing];
x2 = Table[x1[Denominator[[1]], j], {j, 0, ModuloRing - 1}];
x3 = Complement[Table[i, {i, 0, ModuloRing - 1}], x2];
Qx = {};
Nx = Numerator;
NoelUpdate :=
Module[{ux1, ux2, ux3, ux4},
If[MemberQ[x3, Nx[[1]]], Throw[{Qx, Nx, Denominator, False}]];
ux1 = Take[Nx, Length[Denominator]];
ux2 = Position[x2, ux1[[1]] // Flatten // (#[[1]] - 1) &;
ux3 = Mod[ux1 - ux2 Denominator, ModuloRing];
Qx = Join[Qx, {ux2}];
ux4 = Drop[Nx, Length[Denominator]];
Nx = Join[Rest[ux3], ux4];
Catch[Table[NoelUpdate, {i, 1, Length[Numerator] - Length[Denominator] + 1}],
{Qx, Nx, Denominator, True}]
]

DropLeadingZeros[DenominatorSeq_] :=
Module[{x1},
x1[{}] := {};
x1[DS_] := DS[[1]] == 0 := x1[Rest[DS]];
x1[DS_] := DS; x1[DenominatorSeq]
]

PossibleDivisors[ModuloRing_][Order_] :=
Module[{x1, x2, x3, x4},
x1 = Table[{x2[i], 0, ModuloRing - 1}, {i, 0, Order}];
x3 = Table[x2[i], {i, 0, Order}];
x4 = Apply[Table, Sequence[Join[{x3}, x1]]] // Flatten[#, Order] & // Rest;
Map[DropLeadingZeros, x4] ]

PolynomialMultiplication[ModuloRing_][poly1_, poly2_] :=
Module[{x1, x2, x3, x4, x5, x6, x7, x8, x9},
x1 = Length[poly1] + Length[poly2] - 1;
x2 = Table[x3^i, {i, 0, x1 - 1}];
x4 = (Take[x2, Length[poly1]] // Reverse) . poly1;
x5 = (Take[x2, Length[poly2]] // Reverse) . poly2;
x6 = x5 x4;
x7 = Table[Coefficient[x6, x3^(x1 - i - 1)] // Mod[#, ModuloRing] &, {i, 0, x1 - 2}];
x8 = Join[x7, {Mod[x6 /. x3 -> 0, ModuloRing]}] // DropLeadingZeros]

```

We define polynomial multiplication, modulo a primitive polynomial

```

ModuloMultiplication[ModuloRing_][PrimitivePolynomial_][poly1_, poly2_] :=
Module[{x1, x2, x3, x4},
x1 = PolynomialMultiplication[ModuloRing][poly1, poly2];
x2 = RingDivision[ModuloRing][{PrimitivePolynomial, x1}];
x2[[2]] // DropLeadingZeros]

RingPower[RingModulo_][PrimitiveElement_][Element_] :=
Module[{x1, st, NoelUpdate},
x1 = Length[PrimitiveElement] - 1;
st = {1};
NoelUpdate :=
st = ModuloMultiplication[RingModulo][PrimitiveElement][st, Element];
Table[NoelUpdate, {i, 1, 2^x1 - 1}]

```

```

CyclicMultiplativeGroup[ModuloRing_] [PrimitivePolynomial_] :=
Module[{x1, x2, x3, x4, x5, x6, x7, x8, x9},
  x1 = (Length[PrimitivePolynomial] + 1) / 2 // Floor;
  x2 = PossibleDivisors[ModuloRing][x1];
  x3 = Map[RingDivision[ModuloRing][{#, PrimitivePolynomial}]&, x2];
  x4 = Select[x3, #[[4]] == True &];
  x5 = Select[x4, #[[2]] == {1} &] // Transpose // Join[#[[1]], #[[3]]]&;
  x6 = Select[x4, #[[2]] == {ModuloRing - 1} &] // Transpose // Join[#[[1]], #[[3]]]&;
  x7 = Join[{1}, {ModuloRing - 1}], x5, x6] // Union;
  x8 = Map[ModuloMultiplication[ModuloRing][PrimitivePolynomial][#, #]&, x7] // Union;
  x9 = RingPower[ModuloRing][PrimitivePolynomial][x8[[2]]] // RotateRight ]

ZeroSequences[ModuloRing_] [Order_] :=
Module[{x1, x2, x3, x4},
  x1 = Table[{x2[i], 0, 1}, {i, 1, Order}];
  x3 = Table[ModuloRing/2 x2[i], {i, 1, Order}];
  x4 = Apply[Table, Sequence[Join[{x3}, x1]]] // Flatten[#, Order - 1]& ]

ZeroPad[Order_] [Element_] :=
Module[{x1, x2, x3},
  x1 = Table[0, {i, 1, Order - Length[Element]}];
  x2 = Join[x1, Element]]

TwoAdicExpansion[ModuloRing_] [PrimitivePolynomial_] :=
Module[{x1, x2, x3, x4, x5, x6, x7, x8, x9},
  x1 = Length[PrimitivePolynomial] - 1;
  x2 = CyclicMultiplativeGroup[ModuloRing] [PrimitivePolynomial];
  x3 = Map[ZeroPad[x1], x2];
  x4 = Join[{Table[0, {i, 1, x1}]}], x3];
  x5 = Function[x, Mod[x[[1]] + 2 x[[2]], ModuloRing]];
  x6 = Table[{x4[[i]], x4[[j]], x5[{x4[[i]], x4[[j]]}] // DropLeadingZeros},
    {i, 1, Length[x4]}, {j, 1, Length[x4]}] //
  Flatten[#, 1]&]

σ[ModuloRing_] [PrimitivePolynomial_] [TwoAdicElement_] :=
Module[{x1, x2, x3, x4, x5, x6, x7},
  {x1, x2, x3} = TwoAdicElement;
  x4 = Length[PrimitivePolynomial] - 1;
  x5 = ModuloMultiplication[ModuloRing] [PrimitivePolynomial][x1, x1] // ZeroPad[x4];
  x6 = ModuloMultiplication[ModuloRing] [PrimitivePolynomial][x2, x2] // ZeroPad[x4];
  Mod[2 x6 + x5, ModuloRing]]

AutomorphismSigma[ModuloRing_] [PrimitivePolynomial_] :=
Module[{x1, x2, x3, x4, x5, x6, x7, x8, x9},
  x1 = TwoAdicExpansion[ModuloRing] [PrimitivePolynomial];
  x2 = Map[σ[ModuloRing] [PrimitivePolynomial], x1];
  x3 = Map[ZeroPad[Length[PrimitivePolynomial] - 1], Transpose[x1][[3]]];
  {x3, x2} // Transpose]

UnitsRing[ModuloRing_] [PrimitivePolynomial_] :=
Module[{x1, x2, x3, x4},
  x1 = AutomorphismSigma[ModuloRing] [PrimitivePolynomial];
  x2 = x1 // Transpose // #[[1]]&;
  x3 = x2 // Mod[2 #, 4]& // Union;
  Complement[x2, x3]]

UnitsRing[4][{1, 0, 0, 3, 2, 3}];

τ[Order_] [σ_] [x_] := (NestList[σ, x, Order - 2] // Apply[Plus, #]&)

TraceRepresentation[ModuloRing_] [PrimitivePolynomial_] :=
Module[{x1, x2, x3, x4, x5, x6, x7, x8, x9},
  x1 = Length[PrimitivePolynomial] - 1;
  x2 = AutomorphismSigma[ModuloRing] [PrimitivePolynomial];
  Map[{x3[Evaluate[#[[1]]]] := Evaluate[#[[2]]]}&, x2];
  x4 = Map[{#, τ[x1][x3][#]}&, Transpose[x2][[1]]] // Mod[#, ModuloRing]&
]

```

```

TupleRepresentation[ModuloRing_][PrimitivePolynomial_] := Module[{x1, x2, x3},
  x1 = Length[PrimitivePolynomial];
  x2 = NestList[ModuloMultiplication[ModuloRing_][PrimitivePolynomial][{1, 0}, #]&,
    {1}, 2^(x1 - 1) - 2];
  x3 = Map[ZeroPad[x1 - 1], x2]]

```

This algorithm does not always yield the correct value for the Minimum polynomial.

```

MinimumPoly[ModuloRing_][PrimitivePolynomial_][rootPower_][var_] :=
  Module[{x1, x2, x3, x4, x5, x6, x7, x8, x9, x10, x11, x12, x13, x14, x15, tom},
    x1 = Length[PrimitivePolynomial] - 1;
    x2 = 2^x1 - 1;
    x3 = NestList[Mod[#, #, x2]&, rootPower, x2];
    x4 = Position[x3, rootPower] // (#[[2, 1]] - 1)&;
    x6 = Take[x3, x4];
    x7 = TupleRepresentation[ModuloRing_][PrimitivePolynomial];
    x8 = Table[{i, {i, 0, x2 - 1}}];
    x9 = {x8, x7} // Transpose;
    x10 = Map[{var + tom[#]}&, x6] // Apply[Times, #]& // Expand;
    rule = tom[n_] tom[m_] -> tom[n + m];
    x11 = x10 //. rule;
    x12 = Table[Coefficient[x11, var, x1 - i], {i, 0, x1}];
    Map[(tom[Evaluate[#[[1]]]] := Evaluate[#[[2]]])&, x9];
    tom[n_] := Evaluate[tom[Mod[n, x2]]]; Mod[
    x12, ModuloRing]]

GraeffeMethod[GF2minpoly_] :=
  Module[
    {x1, x2odd, x2even, x3odd, x3even, x4even, x4odd, x5, x6even, x6odd, x7, x8, x9},
    x1 = Length[GF2minpoly];
    x2odd = Table[{1, 0}, {i, 1, (x1 + 1) / 2 // Floor}] // Flatten // Take[#, -x1]&;
    x2even = Table[{0, 1}, {i, 1, (x1 + 1) / 2 // Floor}] // Flatten // Take[#, -x1]&;
    x3even = GF2minpoly.x2even // DropLeadingZeros;
    x3odd = GF2minpoly.x2odd // DropLeadingZeros;
    x4even = PolynomialMultiplication[4][x3even, x3even];
    x4odd = PolynomialMultiplication[4][x3odd, x3odd];
    x5 = Max[{Length[x4even], Length[x4odd]}];
    x6even = ZeroPad[x5][x4even];
    x6odd = ZeroPad[x5][x4odd];
    x7 = x6even - x6odd;
    x8 = {x7 Sign[x7[[1]]]} // Mod[#, 4]&;
    Join[x8, {0}] // Partition[#, 2]& // Transpose[#][[1]]&]

SequenceGenerator[ModuloRing_][PrimitivePoly_][InitialCond_][SeqLength_] :=
  Module[{x1, x2State, x3Update, x4},
    x1 = Mod[-Rest[PrimitivePoly], ModuloRing];
    x2State = InitialCond;
    x3Update := Module[{}, x4 = Last[x2State];
      x2State = Join[{Mod[x2State . x1, ModuloRing]}, Drop[x2State, -1]]; x4];
    Table[x3Update, {1, 1, SeqLength}], x2State]

InitialConditions[PrimitivePoly_] := Module[{x1, x2, x3, x4, x5J, x6, x7},
  x1 = Length[PrimitivePoly] - 1;
  x2 = Table[{x3[i], 0, 1}, {i, 1, x1}]; x4 = Table[x3[i], {i, 1, x1}];
  x5J = (Table @@ Sequence[Join[{x4}, x2]]) // Flatten[#, x1 - 1]&;
  x6 = Mod[x5J[[1]] + 2 x5J[[2]], 4];
  x7 = Map[Mod[x5J[[2]] + 2 #, 4]&, x5J];
  Join[x7, {x6}]]

GoldSequence[PrimitivePoly_][InitCondNumber_] := Module[{x1, x2, x3, x4},
  x1 = Length[PrimitivePoly] - 1;
  x2 = InitialConditions[PrimitivePoly][[InitCondNumber]];
  x3 = SequenceGenerator[4][PrimitivePoly][x2][2^x1 - 1];
  x4 = x3[[1]] /. {0 -> 0, 1 -> 0, 2 -> 1, 3 -> 1}]

AutocorrelationSequence[GoldSequence_] :=
  Module[{x1, x2State, x3Update, x4},
    x1 = GoldSequence /. 0 -> -1;
    x2State = x1;
    x3Update := Module[{}, x4 = x1 . x2State; x2State = RotateRight[x2State]; x4];
    Table[x3Update, {1, 1, Length[GoldSequence]}]]

```

```

SpecifiedGoldSequences[PrimitivePoly_] [NumberList_] :=
Module[{x1, x2, x3, x4},
  x1 = Length[PrimitivePoly] - 1;
  x2 = NumberList /. Last -> 2^x1 + 1;
  Map[GoldSequence[PrimitivePoly] [#]&, x2] ]

AllGoldSequences[PrimitivePoly_] :=
Module[{x1, x2, x3, x4},
  x1 = Length[PrimitivePoly] - 1;
  x2 = Table[1, {1, 1, 2^x1 + 1}];
  Map[GoldSequence[PrimitivePoly] [#]&, x2] ]

CrosscorrelationSequence[{GoldSeq1_, GoldSeq2_}] :=
Module[{x1, x2State, x3Update, x4},
  x1 = GoldSeq1 /. 0 -> -1;
  x2State = (GoldSeq2 /. 0 -> -1);
  x3Update := Module[{}, x4 = x1 . x2State; x2State = RotateRight[x2State]; x4];
  Table[x3Update, {1, 1, Length[GoldSeq1]}] ]

End[]

EndPackage[]

```

```

BeginPackage["LaurentFunctions"]

T::usage = "This is the symbol period"

BT::usage = "This is the usual product"

h::usage = "This is the raw gaussian pulse"

B::usage = "This denotes modulation index Pi"

psi::usage = "psi[L,t] is Laurents function"

hFiltered::usage = "This is the filtered gaussian pulse"

PhaseAngle::usage = "This function takes some time to calculate. The following
code will draw a graph of the function PhaseFunction[t_] = N[phi_L,t];
phasepoints = Table[{t, PhaseFunction[t T]} // N, {t, 0, L, 1/40}];
ListPlot[phasepoints, PlotJoined -> True]"

PhaseAngleFast::usage = "PhaseAngleFast[L][t] speeds up the calculation
of PhaseAngle by calculating PhaseAngle[L][t] with L numeric and t
symbolic. It takes some time to calculate."

S::usage = "S = Sin[B]"

J::usage = "J = e^1"

C::usage = "C = Cos[B]"

M::usage = "M = 2^B-1"

ModulationIndex::usage = "ModulationIndex = h"

LaurentS::usage = "LaurentS[L][n][t] = Sin[psi[L,t + n T]] / S"

LaurentC::usage = "LaurentC[L][K][t] is Laurents C_{K,t}"

AlphaKI::usage = "AlphaKI[LL][K,i] is Laurents alpha_{K,i}"

LaurentLK::usage = "LaurentLK[L][K] gives the support of C_{K,t}"

```

#### The start of Modulator Definitions

```

ANKInitialStateSetup::usage =
"ANKInitialStateSetup[L][K][InitBitSeq,AccumulatedPhase] sets up the sequence
of prior states of A_{K,n} that the modulator went through to get to the
constellation point specified by AccumulatedPhase which is really A_{0,0}"

AKN::usage = "AKN[L][K][{State,AccumulatedPhase}] defines A_{K,n} in terms of A_{0,n}"

ModulatingPulse::usage =
"The Pulse is assumed to have the following structure Pulse[L][K][t]"

NumberOfCurves::usage = "The number of pulses used by the modulator"

SamplingInterval::usage =
"SamplingInterval is the interval between samples of the output of the modulator"

InitialState::usage = "InitialState is the set of bits that are assumed
to be present before i.e {a_{-1},a_{-2},...}"

StartingQuadrant::usage = "StartingQuadrant = A_{0,-1} and is a number"

```

### Start of the Receiver Functions

$$h(t_-) := \frac{\text{Exp}\left[-\frac{t^2}{2\sigma^2 T^2}\right]}{\sqrt{2\pi}\sigma T}$$

```
PhaseAngle[L][t_] = Module[{x1, x2, x3, x4, x5, x6}, x1 = hFiltered[3 L T][t1 -  $\frac{L T}{2}$ ];  
x2 = Table[{t2,  $\int_{t_1}^{t_2}$  Evaluate[x1] dt1}, {t2, 0, L T,  $\frac{T}{100}$ ]]; Interpolation[x2][t]
```

```
PhaseAngleFast[L_][t_] := Module[{x1}, x1[tt_] = PhaseAngle[L][tt]; x1[t]];
```

```

ψ[L_, t_] /; 0 < t < LT := PhaseAngleFast[L][t]
ψ[L_, t_] /; 2 LT > t ≥ LT := 0 - PhaseAngleFast[L][t - LT]

```

We need to put this to avoid the function being extrapolated

```

ψ[L_, t_] /; ! (0 < t < LT) && ! (2 LT > t ≥ LT) := 0
LaurentS[L_][n_][t_] := Sin[ψ[L, t + n T]] / 8
AlphaKI[LL_][K_, i_] /; (0 < i < LL) && (0 ≤ K < 2LL-1) :=
Module[{x1, x2, x3, KNum}, x1 := {x2 = Mod[KNum, 2]; KNum =  $\frac{KNum - x2}{2}$ ; x2};
KNum = K; x3 = Table[x1, {i1, 0, LL - 1}]; x3[[i]]
LaurentLK[L_][K_] :=
Module[{x1}, x1 = Table[L (2 - AlphaKI[L][K, i1]) - i1, {i1, 1, L - 1}]; Min[x1]
LaurentC[L_][K_][t_] /; 0 ≤ K < 2L :=
LaurentS[L][0][t]  $\prod_{i1=1}^{L-1}$  LaurentS[L][i1 + L AlphaKI[L][K, i1]][t]

```

The start of the modulator function

```

ANKInitialStateSetup[L_][K_][InitBitSeq_, AccumulatedPhase_] :=
Module[{x1, x2, x3, x4, x5, acuphase, initbitseq},
initbitseq = InitBitSeq;
acuphase = AccumulatedPhase;
UpdateSeq :=
Module[{}, x1 = acuphase - Sum[initbitseq[[i]] AlphaKI[L][K, i], {i, 1, L - 1}];
acuphase = acuphase - First[initbitseq]; initbitseq = Rest[initbitseq]; x1];
Table[UpdateSeq, {i, 1, LaurentLK[L][K]}]
AKN[L_][K_][{State_, AccumulatedPhase_}] :=
AccumulatedPhase - Sum[State[[i + 1]] AlphaKI[L][K, i], {i, 1, L - 1}]
Options[Modulator] := {StartingQuadrant → 0, InitialState → Table[1, {i, 1, 20}],
SamplingInterval → T/32, NumberOfCurves → 4, ModulatingPulse → LaurentC}
Modulator[L_][BitSeq_, Opts_] :=
Module[
{x1, x2, x3, x4, x5, x6, state, AccumulatedPhase, seq, AKNState, Curves, Pulse},
x1 = SamplingInterval /. {Opts} /. Options[Modulator];
state = InitialState /. {Opts} /. Options[Modulator];
x3 = StartingQuadrant /. {Opts} /. Options[Modulator];
x4 = SamplingInterval /. {Opts} /. Options[Modulator];
Pulse = ModulatingPulse /. {Opts} /. Options[Modulator];
Curves = (NumberOfCurves /. {Opts} /. Options[Modulator]) - 1;
AccumulatedPhase = x3;
seq = BitSeq;
Table[
AKNState[K] = ANKInitialStateSetup[L][K][state, AccumulatedPhase], {K, 0, Curves}];
x5 := Module[{}, state = Join[{First[seq]}, Drop[state, -1]];
AccumulatedPhase = AccumulatedPhase + First[seq];
seq = Rest[seq];
Table[AKNState[K] = Join[{AKN[L][K][{state, AccumulatedPhase}],
Drop[AKNState[K], -1]}, {K, 0, Curves}];
x6[t_] = Sum[Sum[{J} AKNState[x][[i+1]] Pulse[L][K][t + i T],
{i, 0, LaurentLK[L][K] - 1}], {K, 0, Curves}];
Table[x6[t], {t, 0, T - x4, x4}];
Table[x5, {kk, 1, Length[BitSeq]}] // Flatten]
Options[Receiver] := {StartingQuadrant → 0,
InitialState → {1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}, SamplingInter
ModulatingPulse → FiltPulse, SyncSample → 0, NumberOfCurves → 2};

```

```

Receiver[L_][InputSeq_, Opts_] :=
Module[{x1, x2, x3, x4, x5, x6},
  x1 = StartingQuadrant /. {Opts} /. Options[Receiver];
  x2 = InitialState /. {Opts} /. Options[Receiver];
  x3 = SamplingInterval /. {Opts} /. Options[Receiver];
  x4 = ModulatingPulse /. {Opts} /. Options[Receiver];
  x5 = SyncSample /. {Opts} /. Options[Receiver];
  x6 = NumberOfCurves /. {Opts} /. Options[Receiver];
ReceiverProper[L][x1, x2, x3, x4, x5, x6, InputSeq]

ReceiverProper[L_][StartingQuadrant_, InitialState_, SamplingInterval_,
ModulatingPulse_, SyncSample_, NumberOfCurves_, InputSeq_] :=
Module[{x1, sgn, ReceivedSeq, ExpectedValue, seq, ReceiveNext, D, N},
  x1 = T / SamplingInterval;
  ReceivedSeq = Partition[InputSeq, x1] // Transpose // #[[SyncSample + 1]] &;
  ExpectedValue =
    ModulatingPulse[L][0][(LaurentLK[L][0] / 2) T + SyncSample SamplingInterval];
  N = StartingQuadrant;
  sgn = 0;
  D = {};
  seq = ReceivedSeq;
  ReceiveNext :=
Module[{x1, x2},
  x1 = ((-1)sgn JN First[seq]) // Im;
  If[Abs[x1 - ExpectedValue] <= Abs[x1 + ExpectedValue],
    D = Join[D, {1}]; N = N + 1; D = Join[D, {-1}]; N = N - 1;
    seq = Rest[seq]; sgn = Mod[sgn + 1, 2];
  Table[ReceiveNext, {i, 1, Length[ReceivedSeq]}];
  D]

End[]

EndPackage[]

```